MATH 101 (Differentiation)

Practice sits and Questions

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EASTERN UNIVERSITY

Practice Sheet #1

Function, Domain & Range

Find out the domain and range of the following functions and also sketch the graph:

1. $f(x)=\dfrac{1}{x-3}$

2. $f(x)=\sqrt{x^2-9}$

3. $f(x)=\sqrt{9-x^2}$

4. $f(x)=\sqrt{x^2-5x+6}$

5. $f(x)=\dfrac{x}{|x|}$

6. $f(x)=x^3+2$

7. $f(x)=\begin{cases} x+2 & ,x\le 1 \\ x^3 & ,|x|<1 \\ -x+3 & ,x\ge 1 \end{cases}$

8. $f(x)=\begin{cases} \dfrac{x^2-1}{x-1} & ,x\ne 1 \\ 2 & ,x=1 \end{cases}$

9. $f(x)=3Sinx$

10. $f(x)=-\sqrt{x^2-7x+10}$

11. $f(x)=\dfrac{2x}{x-4}$

12. $f(x)=\dfrac{1}{5x+7}$

13. $f(x)=\ln(x^2+1)$

14. $f(x)=\begin{cases} x^2 & ,x<0 \\ x & ,0\le x\le 1 \\ \dfrac{1}{x} & ,x>1 \end{cases}$

15. $f(x)=\begin{cases} 2x+6 & ,-3\le x\le 0 \\ 6 & ,0<x<2 \\ 2x-6 & ,2\le x\le 5 \end{cases}$

16. $f(x)=Sin^2x$

17. $f(x)=e^x$

18. $f(x)=Logx$

19. $f(x)=\sqrt{2x+4}$

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Calculus I (MAT-101)
Practice Sheet#2

1. $\lim_{x\to 0} \frac{x}{\sqrt{x+1}-1}$

2. $\lim_{x\to 0} \frac{2x^2-5x+2}{5x^2-7x-6}$

3. $\lim_{x\to 0} \frac{x}{|x|}$

4. $f(x)=\begin{cases} 2-x & ,x<1 \\ x^2+1 & ,x>1 \end{cases}$

   Find $\lim_{x\to 0} f(x)$

5. $f(x)=\begin{cases} \frac{1}{x+2} & ,x<-2 \\ x^2-5 & ,-2<x<3 \\ \sqrt{x+13} & ,x>3 \end{cases}$

   Find $\lim_{x\to 0} f(x)$ and $\lim_{x\to 3} f(x)$

6. $\lim_{x\to\infty} \frac{3x+5}{6x-8}$

7. $\lim_{x\to\infty} \left(\sqrt{x^6+5x^3}-x^3\right)$

8. $\lim_{x\to\infty} \sqrt{\frac{3x+5}{6x-8}}$

9. $\lim_{x\to -\infty} \frac{4x^2-x}{2x^3-5}$

10. $\lim_{x\to\infty} \left(\sqrt{x^6+5}-x^3\right)$

11. $f(x)=\begin{cases} x^2+1 & ,x>0 \\ 1 & ,x=0 \\ 1+x & ,x<0 \end{cases}$

    Find $\lim_{x\to 0} f(x)$

12. $f(x)=\begin{cases} 3x-1 & ,x<1 \\ 3-x & ,x>1 \end{cases}$

    Find $\lim_{x\to 1} f(x)$

13. $f(x)=\begin{cases} x^2 & ,x<1 \\ \text{~~2.4~~} & \text{~~,x = 1~~} \\ x^2+1 & ,x>1 \end{cases}$

    Does $\lim_{x\to 1} f(x)$ exist?

14. $f(x)=\begin{cases} e^{-\frac{|x|}{2}} & ,-1<x<0 \\ x^2 & ,0<x<2 \end{cases}$

    Find $\lim_{x\to 0} f(x)$

15. $f(x)=\begin{cases} 2x+1 & ,x<1 \\ 3-x & ,x>1 \end{cases}$

    Find $\lim_{x\to 1} f(x)$

16. Prove that $\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x = e$

Practice Sheet #3
Continuity and Differentiability

Gulshan Kahtun

(a) Test the continuity of the following functions:

1. $f(x)=\begin{cases}\cos x, & x\geq 0\\ -\cos x, & x<0\end{cases}$ at $x=0$.

2. $f(x)=\begin{cases}x\cos(1/x), & x\neq 0\\ 0, & x=0\end{cases}$ at $x=0$.

3. $f(x)=\begin{cases}e^{1/x}, & x\neq 0\\ 1, & x=0\end{cases}$ at $x=0$.

4. $f(x)=\begin{cases}e^{\frac{-|x|}{2}}, & -1<x<0\\ x^2, & 0\leq x<2\end{cases}$ at $x=0$.

5. $f(x)=\begin{cases}(x-a)\sin\left(\dfrac{1}{x-a}\right), & x\neq a\\ 0, & x=a\end{cases}$ at $x=a$.

6. $f(x)\begin{cases}1 & ,x<0\\ 1+\sin x & ,0\leq x<\pi/2\\ 2+(x-\pi/2)^2 & ,x\geq \pi/2\end{cases}$

at $x=0$ and $x=\pi/2$

7. $f(x)=\begin{cases}x\sin(1/x), & x\neq 0\\ 0, & x=0\end{cases}$ at $x=0$.

8. $f(x)=\begin{cases}\dfrac{|x-3|}{x-3} & , x\neq 3\\ 0 & , x=3\end{cases}$ at $x=3$

9. ~~$f(x)=|x|+|x-1|$ at $x=0$ and $x=1$~~ $f(x)=\begin{cases}1-2x, & x<0\\ 1, & 0\leq x\leq 1\\ 2x-1, & x>1\end{cases}$ at $x=0$ or, $x=1$

~~10.~~ $f(x)=\begin{cases}(1+x)^{1/x} & ,x\neq 0\\ 1 & ,x=0\end{cases}$ at $x=0$. ✗

(b) Test the differentiability of the following functions:

1. $f(x)=\begin{cases}\cos x, & x\geq 0\\ -\cos x, & x<0\end{cases}$ at $x=0$.

2. $f(x)=\begin{cases}x^2\sin(1/x), & x\neq 0\\ 0, & x=0\end{cases}$ at $x=0$.

3. $f(x)=|x|$ at $x=0$

4. $f(x)=\begin{cases}x\cos(1/x), & x\neq 0\\ 0, & x=0\end{cases}$ at $x=0$.

5. Let $f(x)=\begin{cases}x^2-16x & ,x<9\\ 12\sqrt{x} & ,x\geq 9\end{cases}$. Is $f(x)$ continuous at $x=9$? Determine whether $f(x)$ is differentiable at $x=9$.

6. Let $f(x)=\begin{cases}x^2, & x\leq 1\\ \sqrt{x} & ,x>1\end{cases}$. Is $f(x)$ continuous at $x=1$? Determine whether $f(x)$ is differentiable at $x=1$.

7. Show that $f(x)=\begin{cases}x^2+1, & x\leq 1\\ x & ,x>1\end{cases}$ is not continuous and differentiable at $x=1$. Sketch the graph of $f(x)$. graph আঁকো

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Calculus I (MAT-101)
Practice Sheet#4

Indeterminate Forms

Find the limit using L'Hospital rule:

1. $\lim\limits_{x\to 1}\dfrac{\ln x}{x-1}$, 2. $\lim\limits_{x\to 3}\dfrac{x-3}{3x^2-12x+12}$, 3. $\lim\limits_{x\to 0}\left(\dfrac{1}{x^2}-\dfrac{\cos 3x}{x^2}\right)$, 4. $\lim\limits_{x\to \pi}\dfrac{\sin x}{x-\pi}$,

5. $\lim\limits_{x\to 0}\dfrac{x-\tan^{-1}x}{x^3}$, 6. $\lim\limits_{x\to +\infty}\dfrac{e^{3x}}{x^2}$, 7. $\lim\limits_{x\to 0}\left(e^x+x\right)^{1/x}$, 8. $\lim\limits_{x\to 0}\left(\dfrac{1}{x}-\dfrac{1}{\sin x}\right)$, 9. $\lim\limits_{x\to \infty} xe^{-x}$,

10. $\lim\limits_{x\to \pi}(x-\pi)\cot x$, 11. $\lim\limits_{x\to 0}\dfrac{\ln(\sin x)}{\ln(\tan x)}$, 12. $\lim\limits_{x\to 0}\dfrac{\sin 2x}{x}$, 13. $\lim\limits_{x\to 0}\dfrac{\sin x}{x^2}$, 14. $\lim\limits_{x\to \infty}\dfrac{x}{e^x}$,

15. $\lim\limits_{x\to 0}\left(\dfrac{1}{x}+\dfrac{1}{xe^x}\right)$, 16. $\lim\limits_{x\to 0}\dfrac{\sin 2x}{\sin 5x}$.

Rolle's and Mean Value Theorem

1. Verify the hypothesis / Discuss the application of Rolle's Theorem for the following functions:

   a) $f(x)=x^2-6x+8;\ [2,4]$
   b) $f(x)=\cos x;\ \ [\pi/2, 3\pi/2]$
   c) $f(x)=\sqrt{25-x^2};\ \ [0,5]$

2. Verify the hypothesis / Discuss the application of Mean Value Theorem for the following functions:

   a) $f(x)=x^3+x-4;\ [-1,2]$
   b) $f(x)=\sqrt{x+1};\ \ [0,3]$
   c) $f(x)=\sqrt{25-x^2};\ \ [0,5]$

Maclaurin and Taylor Series

1. Find the Taylor Series for the following functions:

   (i) $\sin x$, at $x_0=\pi/2$ (ii) $\ln x$, at $x_0=2$

2. Expand $y=\ln x$, in the power of $x-2$ and $y=e^{ax}$ in the power of $x-1$.

3. Find the Maclaurin Series for the function e^{ax} and $\cos x$.

4. Expand $y=\ln(x+1)$ and $y=\sin x/\cos x$ in the power of x.

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Calculus I (MAT-101)
Practice Sheet#5

Techniques of Differentiation

1. Find the differential coefficients $\left(i.e\ \frac{dy}{dx}\right)$ of the following functions w.r.to x.

(i) $3x^4 - x^2y + 2y^3 = 0$ (ii) $y = \sqrt{\left(\frac{1+x}{1-x}\right)}$ (iii) $x^y = y^x$ (iv) $(\sin x)^{\ln x}$

(iv) $x = \sin^3 t,\ y = \tan t$ (v) $y = \sin x \sin 2x \sin 3x$.

Successive Differentiation

1. Find the $n\,th$ derivative of the following functions:

a) $y = x^n$ b) $y = (ax+b)^n$ c) $y = \ln(ax+b)$ d) $y = 1/(x+a)$ e) $y = \cos(ax+b)$

f) $y = e^x$ g) $y = \sin(ax+b)$

2. If $y = e^{ax}\sin bx$, then show that $y_2 - 2ay_1 + (a^2+b^2)y = 0$.

3. If $y = e^x \sin x$, then show that $y_4 + 4y = 0$.

Leibnitz's Theorem

1. If $y = \tan^{-1} x$, then show that $(1+x^2)y_{n+2} + 2(n+1)x\,y_{n+1} + n(n+1)y_n = 0$.

2. If $y\sqrt{1-x^2} = \sin^{-1} x$, then show that $(1-x^2)y_{n+1} - (2n+1)x\,y_n - n^2\,y_{n-1} = 0$.

3. If $y = e^{\tan^{-1} x}$, then show that $(1+x^2)y_{n+2} + (2nx+2x-1)x\,y_{n+1} + n(n+1)y_n = 0$.

4. If $y = (\sin^{-1} x)^2$, then show that $(1-x^2)y_{n+2} - (2n+1)x\,y_{n+1} - n^2 y_n = 0$.

5. If $y = e^{m\sin^{-1} x}$, then show that $(1-x^2)y_{n+2} - (2n+1)x\,y_{n+1} - (n^2+m^2)y_n = 0$.

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MAT-117
Differential and Integral Calculus

Maximum & Minimum

Process of determining the maximum and minimum value of a function:

Let f(x) is a function. To determine the maximum and minimum value of the function we may follow the following steps-

Step-1: find $f'(x)$

Step-2: to determine the turning points solve it for $f'(x) = 0$. Let the solution is α

Step-3: if $f''(\alpha)$ gives +'ve value then the function has a minimum value for α.

If $f''(\alpha)$ gives -'ve value then the function has a maximum value for α.

Step-4: then to find the maximum or minimum value of the function find $f(\alpha)$

Problems:

Find the Maximum & Minimum value of the functions (1-14)-

1. $f(x) = 2x^3 - 21x^2 + 36x - 20$
2. $f(x) = 5x^6 - 18x^5 + 15x^4 - 10$
3. $f(x) = x^3 - 9x^2 + 24x - 12$
4. $f(x) = 4x^3 - 15x^2 + 12x - 2$
5. $f(x) = x^3 - 3x^2 - 9x$
6. $f(x) = 2x^3 - 9x^2 + 12x - 3$
7. $f(x) = x^6 - 5x^4 + 5x^3 - 1$
8. $f(x) = 1 + 2\sin x + 3\cos^2 x \quad \left(0 \le x \le \frac{\pi}{2}\right)$
9. $f(x) = x^{\frac{1}{x}}$
10. $f(x) = 17 - 15x + 9x^2 - x^3$
11. $f(x) = x^3 - 6x^2 + 9x + 5$
12. $f(x) = 2x^3 - 9x^2 + 12x - 3$
13. $f(x) = x^3 - 3x^2 - 9x$
14. $f(x) = x^3 - 6x^2 + 12x - 3$

Assignment # 1

Maxima and minima

Last date of submission 18/02/08

1. Find (a) the open intervals on which f is increasing, (b) the open intervals on which f is decreasing, (c) the open intervals on which f is concave up, (d)) the open intervals on which f is concave down and (e) the x- coordinate of all inflection points.

   $(i)\ f(x)=x^2-5x+6$ $(ii)\ f(x)=5+12x-x^3$

   $(iii)\ f(x)=x^4-8x^2+16$ $(iv)\ f(x)=\dfrac{x^2}{x^2+2}$ $(v)\ f(x)=\sqrt[3]{x+2}.$

   $(vi)\ f(x)=Cosx;\ [0,2\pi]$ $(vii)\ f(x)=\tan x;\ [-\frac{\pi}{2},\frac{\pi}{2}]$

2. Locate the critical numbers and identify which critical numbers correspond to stationary points.

   $(i)\ f(x)=x^3+3x^2-9x+1$ $(ii)\ f(x)=x^4-6x^2-3$

   $(iii)\ f(x)=\dfrac{x}{x^2+2}$ $(iv)\ f(x)=x^{2/3}$

   $(v)\ f(x)=x^{1/3}(x+4)$ $(vi)\ f(x)=\cos 3x.$

3. Find the relative extrema (maxima/ minima) using both the first and second derivative tests.

   $(i)\ f(x)=2x^3-9x^2+12x$ $(ii)\ f(x)=\dfrac{x}{2}-\sin x,\quad 0<x<2\pi.$

4. Use the given derivative to find all critical numbers of f and determine whether a relative maximum, relative minimum, or neither occurs there.

   $(i)\ f'(x)=x^3(x^2-5)$ $(ii)\ f'(x)=\dfrac{x^2-1}{x^2+1}.$

Eastern University
Faculty of Engineering & Technology
B.Sc. Engg. Mid Term Exam, Spring- 2011
Course Name: Calculus I (Day)
Course No. MAT-101/117(Group 1 & 3)

Total Time: 90 Minutes **Total Marks: 30**

NB. 1. Answer any three questions.

2. The right margin indicates the marks associated with each question.

1.a) Verify the Rolle's theorem for the function $f(x) = x^3 - x^2 - 4x + 4$ in the interval $(-2, 2)$. [4]

b) A function is defined as follows: [6]

$$f(x) = \begin{cases} 2x+6 & ,-2 \le x \le 0 \\ 8 & ,0 < x < 2 \\ 2x-6 & ,3 \le x \le 7 \end{cases}$$

Sketch the graph and find the domain & range of the function.

2.a) What do you mean by differentiability? [2]

b) Test the Differentiability of the function: [4]

$$f(x) = \begin{cases} x^2 \sin(1/x) & ;x \ne 0 \\ 0 & ;x = 0 \end{cases} \quad \text{at } x = 0.$$

c) Evaluate the following limit: $\lim_{x \to 0} \left(e^x + x\right)^{1/x}$ (Using L'Hospital Rule). [4]

3.a) Evaluate the following limit if it exists- [4]

$$\lim_{x \to 1/2} f(x) \text{ where } f(x) = \begin{cases} 2x+1 & ,1/2 \le x < 0 \\ 1-2x & ,0 \le x < 1/2 \\ 2x-1 & ,x > 1/2 \end{cases}$$

b) Find the relative extremum value (Maximum & Minimum value) of the function $f(x) = 2x^3 - 21x^2 + 36x - 20$. [6]

4.a) Define i) function & ii) limit with example. [3]

b) State Leibnitz's Theorem. [2]

c) If $y=\left(cos^{-1}x\right)^2$, then show that, [5]

$\left(1-x^2\right)y_{n+2}-\left(2n+1\right)x\,y_{n+1}-n^2y_n=0$. (using Leibnitz's Theorem)

Eastern University
B.Sc. Engg. Quiz#2, Spring- 2011
Course Name: Calculus I(MAT 101) (Day & Group 3)

Total Time: 30 minutes **Total Marks: 20**
Answer all the questions.

1. i) Define the continuity? [2+2]
   ii) State the Rolle's theorem.
2. Test the Differentiability of the function: [5]

   Let $f(x)=\begin{cases} x^2-16x, & x<9 \\ 12\sqrt{x}, & x\geq 9 \end{cases}$ at $x=9$

3. Verify the Mean Value theorem for the function $f(x)=x-x^3$ in the interval $(-2,1)$. [5]

4. If $y=e^{2Sin^{-1}x}$, then using Leibnitz's theorem show that, [6]

   $(1-x^2)y_{n+2}-(2n+1)x\,y_{n+1}-(n^2+4)y_n=0$.

   OR

   If $y=e^x \sin x$, then show that $y_4+4y=0$.

Eastern University
B.Sc. Engg. Quiz#1, Spring- 2011
Course Name: Calculus I(MAT 101) (Day & Group 3)

Total Time: 30 minutes **Total Marks: 20**
Answer all the questions.

1. Define domain and range of a function with example. [3]
2. A function is defined as follows: [6]

   $$f(x)=\begin{cases} x^2, & x<0 \\ x, & 0\leq x\leq 1 \\ \dfrac{6}{x}, & x>1 \end{cases}$$

   Sketch the graph and find the domain & range of the function.

3. Evaluate the following limit if it exists- [4]

   $$\lim_{x\to 0} f(x) \text{ where } f(x)=\begin{cases} x^2+1, & x>0 \\ 1, & x=0 \\ 1+x, & x<0 \end{cases}$$

4. Evaluate the following limits: i) $\lim_{x\to 0}\dfrac{xe^x-\log(1+x)}{x^2}$ ii) $\lim_{x\to 0}(\sin x)^{\tan x}$ [3+4]

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