
# MATH 101 (Differentiation)

## Practice sits and Questions

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# Formulas

## Algebraic Formulas

1. $(x+a)^n = x^n + {}^nC_1 x^{n-1}a + {}^nC_2 x^{n-2}a^2 + \ldots\ldots\ldots\ldots + {}^nC_n a^n.$

2. $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots\ldots\ldots$

3. $\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \ldots\ldots\ldots\ldots$

4. $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots\ldots\ldots\ldots$

5. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\ldots\ldots\ldots$

6. $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots\ldots\ldots\ldots$

7. $\tan x = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} \ldots\ldots\ldots\ldots$

8. $(1-x)^{-1} = 1 + x + x^2 + x^3 + \ldots\ldots\ldots\ldots$

9. $(1+x)^{-1} = 1 - x + x^2 - x^3 + \ldots\ldots\ldots\ldots$

10. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots\ldots\ldots$

11. $e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots\ldots\ldots\ldots$

## Formulas for Differentiation

1. $\frac{d}{dx}(c) = 0,\ c = \text{constant}$
2. $\frac{d}{dx}\left(x^n\right) = nx^{n-1},$
3. $\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$
4. $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx},\ u = u(x),\ v = v(x)$
5. $\frac{d}{dx}(uv) = u\frac{d}{dx}(v) + v\frac{d}{dx}(u)$
6. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{d}{dx}(u) - u\frac{d}{dx}(v)}{v^2}$
7. $\frac{d}{dx}\left(e^x\right) = e^x$
8. $\frac{d}{dx}\left(e^{mx}\right) = me^{mx}$
9. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
10. $\frac{d}{dx}\left(a^x\right) = a^x \log_e a$
11. $\frac{d}{dx}(\log_a x) = \frac{1}{x}\log_a e$
12. $\frac{d}{dx}(\sin x) = \cos x$
13. $\frac{d}{dx}(\cos x) = -\sin x$
14. $\frac{d}{dx}(\tan x) = \sec^2 x$
15. $\frac{d}{dx}(\sec x) = \sec x \tan x$
16. $\frac{d}{dx}(\cot x) = -\cos ec^2 x$
17. $\frac{d}{dx}(\cos ecx) = -\cos ecx \cot x$
18. $\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$
19. $\frac{d}{dx}\left(\cos^{-1} x\right) = -\frac{1}{\sqrt{1-x^2}}$
20. $\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$
21. $\frac{d}{dx}\left(\cot^{-1} x\right) = -\frac{1}{1+x^2}$
22. $\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{x\sqrt{x^2-1}}$
23. $\frac{d}{dx}\left(\cos ec^{-1} x\right) = -\frac{1}{x\sqrt{x^2-1}}$
24. $\frac{d}{dx}(\sinh x) = \cosh x$
25. $\frac{d}{dx}(\cosh x) = \sinh x$
26. $\frac{d}{dx}(\tanh x) = \sec h^2 x$
27. $\frac{d}{dx}(\sec hx) = -\sec hx \tanh x$
28. $\frac{d}{dx}(\coth x) = -\cos ech^2 x$
29. $\frac{d}{dx}(\cos echx) = -\cos exhx \coth x.$

# Formulas for Integration

**Basic formulas:**

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$
2. $\int \sin x dx = -\cos x + c$
3. $\int \cos x dx = \sin x + c$
4. $\int \sec^2 x dx = \tan x + c$
5. $\int \sec x dx = \ln|\sec x + \tan x| + c$
6. $\int \cos ecx dx = \ln|\cos ecx - \cot x| + c$
7. $\int \tan x dx = \ln|\sec x| + c$
8. $\int \cot x dx = \ln|\sin x| + c$
9. $\int \cos ec^2 x = -\cot x + c$
10. $\int \sec x \tan x dx = \sec x + c$
11. $\int \cos ecx \cot x dx = -\cos ecx + c$
12. $\int \frac{1}{x} dx = \ln x + c$
13. $\int e^{mx} dx = \frac{e^{mx}}{m} + c$
14. $\int a^x dx = \frac{a^x}{\ln x} + c$
15. $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$
16. $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$
17. $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c$

**Formula for integration by parts:**

1. $\int uvdx = u\int vdx - \int\left\{\frac{d}{dx}(u)\int vdx\right\}dx$
2. $\int udv = uv - \int vdu$

**Reduction Formulas:**

1. $\int \sin^n x dx = -\frac{1}{n}\sin^{n-1} x \cos x + \frac{n-1}{n}\int \sin^{n-2} x dx$
2. $\int \cos^n x dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x dx$
3. $\int \tan^n x dx = \frac{1}{n-1}\tan^{n-1} x - \int \tan^{n-2} x dx$
4. $\int \sec^n x dx = \frac{1}{n-1}\sec^{n-2} x \tan x + \frac{n-2}{n-1}\int \sec^{n-2} x dx.$

**Integration of Rational functions:**

1. $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c,$
2. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + c,$
3. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + c,$
4. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$
5. $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c.$

# Formulas

## Algebra Formulas

1. $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots$
2. $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots\ldots$
3. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\ldots$
4. $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots\ldots$
5. $(1-x)^{-1} = 1 + x + x^2 + x^3 + \ldots\ldots$
6. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots$

## Formulas for Differentiation

1. $\frac{d}{dx}(x^n) = nx^{n-1}$
2. $\frac{d}{dx}(e^x) = e^x$
3. $\frac{d}{dx}(e^{mx}) = me^{mx}$
4. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
5. $\frac{d}{dx}(a^x) = a^x \log_e a$
6. $\frac{d}{dx}(\log_a x) = \frac{1}{x}\log_a e$
7. $\frac{d}{dx}(\sin x) = \cos x$
8. $\frac{d}{dx}(\cos x) = -\sin x$
9. $\frac{d}{dx}(\tan x) = \sec^2 x$
10. $\frac{d}{dx}(\sec x) = \sec x \tan x$
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12. $\frac{d}{dx}(\cos ecx) = -\cos ecx \cot x$
13. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
14. $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$
15. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
16. $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$
17. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$
18. $\frac{d}{dx}(\cos ec^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$
19. $\frac{d}{dx}(uv) = u\frac{d}{dx}(v) + v\frac{d}{dx}(u)$
20. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{d}{dx}(u) - u\frac{d}{dx}(v)}{v^2}$

Length →

(A) Kartesio Eqn →

1. $x$ - axis,

$$L = \int_{x=a}^{b} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

2. $y$ - axis,

$$L = \int_{y=c}^{d} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

(B) Parametric Eqn →

1. $x$ - axis & $y$ - axis

$$L = \int_{t=a}^{b} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

(C) Polar Eqn →

$$L = \int_{\theta=\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

Area → (Revolution → Surface)

(A) Kartesio Eqn

1. x-axis,

$$S = \int_{x=a}^{b} 2\pi f(x) \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

2. y-axis,

$$S = \int_{y=c}^{d} 2\pi g(y) \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

(B) Parametric Eqn

1. x-axis,

$$S = \int_{t=a}^{b} 2\pi x \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

2. y-axis,

$$S = \int_{t=c}^{d} 2\pi x \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

(C) Polar Eqn

$$S = \int_{\theta=\theta_1}^{\theta_2} 2\pi r \sin\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

## Volume → (Disks & Washers)

### (A) Disks (2)

1. $x$-axis,

$$V = \int_{x=a}^{b} \pi y^2 \, dx$$

2. $y$-axis,

$$V = \int_{y=c}^{d} \pi x^2 \, dy$$

### (B) Washers (2)

1. $x$-axis,

$$V = \int_{x=a}^{b} \pi \left\{ [f(x)]^2 - [g(x)]^2 \right\} dx$$

2. $y$-axis,

$$V = \int_{y=c}^{d} \pi \left\{ [u(y)]^2 - [v(y)]^2 \right\} dy$$

## Volume → (Cylindrical)

1. $y$-axis,

$$V = \int_{x=a}^{b} 2\pi x [f(x) - g(x)] \, dx$$

2. $x$-axis,

$$V = \int_{y=c}^{d} 2\pi x [u(y) - v(y)] \, dy$$

Reduction Formula →

1. $\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\,dx$

2. $\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x\,dx$

3. $\int \sin^m x \cos^n x\,dx$

- $(m \to \text{odd}) \to$ split $\sin^2 x = 1-\cos^2 x$ & substitute $u = \cos x$

- $(n \to \text{odd}) \to$ split $\cos^2 x = 1-\sin^2 x$ & substitute $u = \sin x$

- $(m, n \to \text{even}) \to$ split $2\sin^2 x = 1-\cos 2x$ & $2\cos^2 x = 1+\cos 2x$

4. $\int \sec^m x \tan^n x\,dx$

- $(m \to \text{odd}) \to$ split $\tan^2 x = 1-\sec 2x$ & substitute $u = \sec x$

- $(n \to \text{odd}) \to$ split $\sec^2 x = 1+\tan^2 x$ & substitute $u = \tan x$

Trigonometric Rules →

1. $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

2. $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

3. $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

4. $2\cos A\cos B = \cos(A-B) + \cos(A+B)$