(GKH)

Ex 6·1

Ex 1. $f(x) = \sqrt{x}$, $[a,b] = [0,1]$; $n = 2$.

Sol^n: $n = 2$, $\Delta x = \frac{1}{2} = .5$.

Let, $n = 10$, $\Delta x = \frac{b-a}{n} = \frac{1-0}{10} = .1$.

$$x_k = x_0 + k \cdot \Delta x$$

$$x_1 = 0 + 1 \cdot (.5) = .5$$

$$x_2 = 0 + 2 \cdot (.5) = 1.$$

$$A_n = \sum_{k=1}^{n} f(x_k) \cdot \Delta x$$

$$= \sum_{k=1}^{2} \sqrt{x_k} \cdot \Delta x$$

$$= \sqrt{.5} \cdot (.5) + \sqrt{1} \cdot (.5)$$

$$= 0.85355339$$

Ans:

Ex 3. $f(x) = \sin x$; $[a,b] = [0,\pi]$.

Sol^n: $n = 2$.

$$\Delta x = \frac{b-a}{n} = \frac{\pi - 0}{2} = \pi/2.$$

$$x_k = x_0 + k \cdot \Delta x$$

$$x_1 = \pi/2$$

$$x_2 = \pi.$$

$$\therefore A_n = \sum_{k=1}^{n} f(x_k) \cdot \Delta x$$

$$= \sum_{k=1}^{2} \sin x_k \cdot \Delta x.$$

$= \sin x_1 \cdot \Delta x + \sin x_2 \cdot \Delta x$

$= \sin \pi/2 \cdot \pi/2 + \sin \pi \cdot \pi/2$

$= \pi/2.$

Ans:

Ex 4. $f(x) = \cos x$; $[a, b] = [0, \pi/2]$; $n = 2, 5$.

Soln: when $n = 2$, $\Delta x = \pi/2 / 2 = \pi/4$.

For right end point, $x^*_k = a + k\Delta x$

$= 0 + k \cdot \pi/4 = \pi/4 \cdot k$.

$f(x^*_k) = \cos(\pi/4 \cdot k)$.

$x_2 = x_0 + 2\Delta x$

$= 0 + 2\frac{\pi}{4}$

$= \frac{2\pi}{4}$

$A_n = \sum_{k=1}^{n} f(x^*_k) \cdot \Delta x$

$= \sum_{k=1}^{2} \cos(k \cdot \pi/4) \cdot \Delta x$

$= \cos \pi/4 \cdot \pi/4 + \cos \frac{2\pi}{4} \cdot \frac{\pi}{4}$

$x_1 = x_0$

$= \pi/4 \left(\cos \pi/4 + \cos \pi/2 \right)$

$= 0.720086474$.

$n = 5$, $\Delta x = \pi/2/5 = \pi/10$.

$x^*_k = a + k\Delta x = k \cdot \pi/10$.

$f(x^*_k) = \cos(k \cdot \pi/10)$.

$$A_n = \sum_{k=1}^{n} f(x^*_k)\cdot \Delta x$$

$$= \sum_{k=1}^{5} \cos(k\cdot \pi/10)\cdot \Delta x$$

$$= \pi/10\left(\cos \pi/10 + \cos \pi/5 + \cos 3\pi/10 + \cos 2\pi/5 + \cos \pi/2\right)$$

$$= 0.957605934.$$

Ans:

Ex 5. $f(x) = \frac{1}{x}$. $[a,b] = [1,2]$; $n = 2$.

soln: $n = 2$, $\Delta x = \frac{2-1}{2} = \frac{1}{2} = .5$.

$$x_k = x_0 + k\cdot \Delta x$$

$$x_1 = 1 + (.5) = 1.5.$$

$$x_2 = 1 + 2(.5) = 2$$

$$A_n = \sum_{k=1}^{n} f(x_k)\cdot \Delta x$$

$$= \sum_{k=1}^{2} \frac{1}{x_k}\cdot \Delta x$$

$$= \frac{1}{x_1}\cdot \Delta x + \frac{1}{x_2}\cdot \Delta x$$

$$= \frac{1}{1.5}(.5) + \frac{1}{2}(.5)$$

$$= \frac{1}{3} + \frac{.5}{2}$$

$$= \frac{3.5}{6}$$

$$= .58333.$$

Ans:

Ex 7. $f(x) = \sqrt{1-x^2}$; $[a,b] = [0,1]$.

soln: when $n=2$, $\Delta x = 1/2$.

$$x^*_k = a + k \cdot \Delta x = 0 + k/2 = k/2.$$

$$\therefore f(x^*_k) = \sqrt{1 - k^2/4} = \sqrt{\frac{4-k^2}{4}} = \frac{\sqrt{4-k^2}}{2}$$

$$\therefore A = \sum_{k=1}^{2} \frac{\sqrt{4-k^2}}{2} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{4-1}}{2} \cdot \frac{1}{2} + \frac{\sqrt{4-4}}{2} \cdot \frac{1}{2}$$

$$= 0.4330127.$$

when $n=5$, $\Delta x = 1/5$.

$$x^*_k = k/5.$$

$$f(x^*_k) = \sqrt{1 - k^2/25}$$

$$= \frac{\sqrt{25-k^2}}{5}$$

$$\therefore A = \sum_{k=1}^{5} \frac{\sqrt{25-k^2}}{5} \cdot \frac{1}{5}$$

$$= \frac{\sqrt{25-1}}{5} \cdot \frac{1}{5} + \frac{\sqrt{25-4}}{5} \cdot \frac{1}{5} + \frac{\sqrt{25-9}}{5} \cdot \frac{1}{5} + \frac{\sqrt{25-16}}{5} \cdot \frac{1}{5} + \frac{\sqrt{25-25}}{5 \times 5}$$

$$= 0.6592622.$$

Ans.

Ex 9. $f(x) = 3$; $[a, x] = [1, x]$.

Sol^n:

$$A(x) = \int_a^x f(x)\,dx$$

$$= \int_1^x 3\,dx = 3\,[x]_1^x = 3(x-1).$$

$$\therefore A'(x) = 3 = f(x)$$

Ans:

Ex 10. $f(x) = 5$; $[a, x] = [2, x]$.

Sol^n: $A(x) = \int_a^x f(x)\,dx$

$$= \int_2^x 5\,dx$$

$$= 5\,[x]_2^x$$

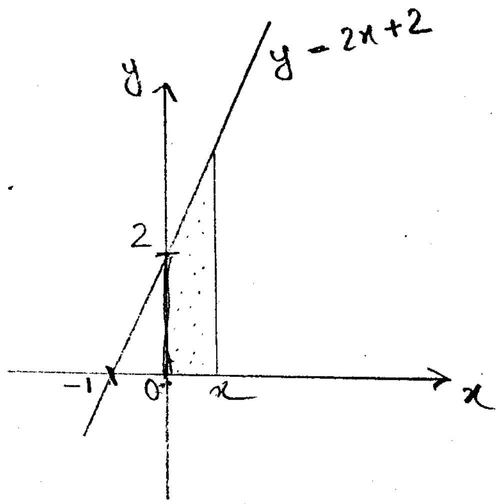
$$= 5(x-2). = 5x - 10.$$

$$\therefore A'(x) = 5 = f(x).$$

Ans:

Ex 11. $y = f(x) = 2x+2$; $[a, x] = [0, x]$.

x	0	-1
y	2	0



The required area is of a trapezoid. If A is the area, then,

Area of trapezoid = ½ (sum of parallel sides) × height.

$$A(x) = \frac{1}{2}[(2x+2)+2]\,x$$

$$= \frac{1}{2}(2x+4)x = x^2 + 2x$$

For this function

$$A'(x) = 2x+2 = f(x)$$

Ans:

Ex - 6.2

Evaluate

Indefinite Integals

19. $\int \left(\frac{2}{x} + 3e^x\right) dx$

$= 2\int \frac{dx}{x} + 3\int e^x dx$

$= 2\ln|x| + 3e^x + c$

Ans:

20. $\int \left(\frac{1}{2t} - \sqrt{2}\, e^t\right) dt$

$= \frac{1}{2}\int \frac{dt}{t} - \sqrt{2}\int e^t dt$

$= \frac{1}{2}\ln|t| - \sqrt{2}\, e^t + c$

Ans:

21. $\int (4\sin x + 2\cos x)\, dx$

$= \int 4\sin x\, dx + \int 2\cos x\, dx$

$= -4\cos x + 2\sin x + c$

Ans:

22. $\int (4\sec^2 x + \operatorname{cosec} x \cot x)\, dx$

$= 4\int \sec^2 x\, dx + \int \operatorname{cosec} x \cot x\, dx$

$= 4\tan x - \operatorname{cosec} x + c$

Ans:

23. $\int \sec x(\sec x + \tan x)\, dx$

$= \int \sec^2 x\, dx + \int \sec x \cdot \tan x\, dx$

$= \tan x + \sec x + c$

Ans:

24. $\int \sec x(\tan x + \cos x)\,dx$

$= \int \left(\sec x \tan x + \cancel{\sec} x \cdot \frac{1}{\cancel{\sec} x}\right) dx$

$= \int \sec x \tan x \,dx + \int dx$

$= \sec x + x + c$

Ans:

25. $\int \frac{\sec\theta}{\cos\theta}\,d\theta$

$= \int \sec\theta \cdot \sec\theta \,d\theta$

$= \int \sec^2\theta \,d\theta$

$= \tan\theta + c.$

Ans:

26. $\int \frac{dy}{\operatorname{cosec} y}$

$= \int \sin y \,dy$

$= -\cos y + c$ Ans:

27. $\int \frac{\sin x}{\cos^2 x}\,dx$

$= \int \tan x \sec x \,dx$

$= \sec x + c.$

Ans:

28. $\int \left(\varphi + \frac{2}{\sin^2\varphi}\right) d\varphi$

$= \int \varphi \, d\varphi + 2\int \frac{d\varphi}{\sin^2\varphi}$

$= \frac{\varphi^2}{2} + 2\int \operatorname{cosec}^2\varphi \, d\varphi$

$= \frac{\varphi^2}{2} + 2(-\cot\varphi) + c$

$= \frac{\varphi^2}{2} - 2\cot\varphi + c$

Ans:

29. $\int (1 + \sin^2\theta \cos\theta) \, d\theta$

$= \int d\theta + \int \sin^2\theta \cos\theta \, d\theta$

Let,
$\sin\theta = u$
$\cos\theta \, d\theta = du$

$= \theta + \int u^2 du$

$= \theta + \frac{u^3}{3} + c$

$= \theta + \frac{\sin^3\theta}{3} + c$

Ans:

30. $\int \frac{\sin 2x}{\cos x} dx$

$= \int \frac{2\sin x \cancel{\cos x}}{\cancel{\cos x}} dx$

$= +2\int \sin x \, dx$

$= -2\cos x + c$

Ans:

Ex: 6.3

Evaluate the integrals by substitutions.

18. $\int \frac{x}{\sqrt{4-5x^2}}\,dx$

Let, $p = 4-5x^2$

$\Rightarrow \frac{dp}{dx} = -10x$

$\therefore x\,dx = \frac{-dp}{10}$

$= \frac{-1}{10}\int \frac{dp}{\sqrt{p}}$

$= \frac{-1}{10}\,\frac{p^{-1/2+1}}{-1/2+1} + c$

$= \frac{-1}{10} \times 2\,(4-5x^2)^{1/2} + c = -\frac{1}{5}(4-5x^2)^{1/2} + c$

Ans:

19. $\int \frac{x^2}{\sqrt{x^3+1}}\,dx$

Let,

$x^3+1 = p$

$\Rightarrow 3x^2\,dx = dp$

$\therefore x^2\,dx = dp/3$

$= \int \frac{dp}{3\sqrt{p}}$

$= \frac{1}{3}\,\frac{p^{-1/2+1}}{-1/2+1} + c$

$= \frac{2}{3}(x^3+1)^{1/2} + c$ Ans:

20. $\int \frac{1}{(1-3x)^2}\,dx$

Let, $1-3x = p$

$\Rightarrow dx = -\frac{dp}{3}$

$= \frac{-1}{3}\int \frac{dp}{p^2}$

$= \frac{-1}{3}\,\frac{p^{-2+1}}{-2+1} + c$

$= \frac{1}{3}\,\frac{1}{(1-3x)} + c$ Ans:

21. $\int \frac{x}{(4x^2+1)^3}\,dx$

Let, $4x^2+1 = p$

$\Rightarrow 8x\,dx = dp$

$\therefore x\,dx = dp/8$.

$= \frac{1}{8}\int \frac{dp}{p^3}$

$= \frac{1}{8}\,\frac{p^{-3+1}}{-3+1} + c$

$= \frac{1}{16(4x^2+1)^2} + c$ Ans:

22. $\int x\cos(3x^2)\,dx$.

Let, $p = 3x^2 \Rightarrow \frac{dp}{dx} = 6x$

$\therefore x\,dx = \frac{dp}{6}$.

$= \frac{1}{6}\int \cos p\,dp$

$= \frac{1}{6}\sin p + c$

$= \frac{1}{6}\sin(3x^2) + c$. Ans:

23. $\int e^{\sin x}\cos x\,dx$.

Let, $p = \sin x$

$\therefore dp = \cos x\,dx$.

$= \int e^p\,dp$

$= e^p + c$

$= e^{\sin x} + c$ Ans:

24. $\int x^3 e^{x^4}\,dx$

Let, $p = x^4$

$\Rightarrow \frac{dp}{dx} = 4x^3$

$\Rightarrow x^3\,dx = \frac{dp}{4}$

$= \frac{1}{4}\int e^p\,dp$

$= \frac{1}{4}e^p + c$

$= \frac{1}{4}e^{x^4} + c$ Ans:

25. $\int x^2 e^{-2x^3} dx$

Let, $P = -2x^3$

$\Rightarrow \frac{dP}{dx} = -6x^2$

$\therefore x^2 dx = \frac{dP}{-6}$

$= \int e^P \frac{dP}{-6}$

$= -\frac{1}{6} e^P + C = \frac{-1}{6} e^{-2x^3} + C$

Ans:

26. $\int \frac{e^x + e^{-x}}{e^x - e^{-x}} dx$

Let, $P = e^x - e^{-x}$

$\Rightarrow \frac{dP}{dx} = e^x + e^{-x}$

$\therefore dP = (e^x + e^{-x}) dx$

$= \int \frac{dP}{P}$

$= \ln|P| + C$

$= \ln|e^x - e^{-x}| + C$ Ans:

27. $\int \frac{e^x}{1+e^{2x}} dx$

Let,

$P = e^x$

$\therefore dP = e^x dx$

Here, $e^{2x} = (e^x)^2$

$= \int \frac{dP}{1+P^2}$

$= \tan^{-1} P + C$

$= \tan^{-1}(e^x) + C$ Ans:

~~28~~. $\int \frac{t}{t^4+1} dt$

Let, $P = t^2$

$\therefore t\,dt = dP/2$.

$= \frac{1}{2} \int \frac{dP}{1+P^2}$

$= \frac{1}{2} \tan^{-1} P + C$

$= \frac{1}{2} \tan^{-1}(t^2) + C$ Ans:

29. $\int \frac{\sin(5/x)}{x^2}\,dx$

$P = 5/x$

$\Rightarrow \frac{dP}{dx} = -5x^{-2} = \frac{-5}{x^2}$

$\therefore \frac{dx}{x^2} = \frac{dP}{-5}$

$= \int \frac{\sin P}{-5}\,dP$

$= \frac{-1}{5}(-\cos P) + C$

$= \frac{1}{5}\cos(5/x) + C$ Ans:

30. $\int \frac{\sec^2\sqrt{x}}{\sqrt{x}}\,dx$

$P = \sqrt{x} \Rightarrow \frac{dP}{dx} = \frac{1}{2\sqrt{x}}$

$\therefore 2dP = \frac{dx}{\sqrt{x}}$

$= 2\int \sec^2 P\,dP$

$= 2\tan P + C = 2\tan\sqrt{x} + C$

Ans:

31. $\int x^2\sec^2(x^3)\,dx$

Let, $P = x^3$

$\Rightarrow \frac{dP}{dx} = 3x^2$

$\therefore \frac{dP}{3} = x^2 dx$.

$= \frac{1}{3}\int \sec^2 P\,dP$

$= \frac{1}{3}\tan P + C$

$= \frac{1}{3}\tan(x^3) + C$ Ans:

32. $\int \cos 4\theta\sqrt{2-\sin 4\theta}\,d\theta$

Let,

$P = 2-\sin 4\theta$

$\Rightarrow \frac{dP}{d\theta} = -\cos 4\theta \times 4$

$\Rightarrow \frac{-dP}{4} = \cos 4\theta\,d\theta$

$= -\frac{1}{4}\int \sqrt{P}\,dP$

$= \frac{-1}{4}\,\frac{P^{3/2}}{3/2} + C$

$= \frac{-1}{6}(2-\sin 4\theta)^{3/2} + C$

Ans:

32. $\int \cos^3 2t \sin 2t \, dt$

Let,

$\cos 2t = p$

$\Rightarrow -2\sin 2t = \frac{dp}{dt}$

$\therefore \sin 2t \, dt = \frac{-dp}{2}$

$= \frac{-1}{2} \int p^3 \, dp$

$= \frac{-1}{2} \frac{p^4}{4} + c$

$= \frac{-1}{8} \cos^4 2t + c.$ Ans:

33. $\int \sin^5 3t \cos 3t \, dt$

Let,

$\sin 3t = p$

$\Rightarrow 3\cos 3t = \frac{dp}{dt}$

$\Rightarrow \cos 3t \, dt = \frac{dp}{3}$

$= \frac{1}{3} \int p^5 \, dp$

$= \frac{1}{3} \frac{p^6}{6} + c$

$= \frac{1}{18} \sin^6 3t + c.$ Ans:

34. $\int \frac{\sin 2\theta}{(5+\cos 2\theta)^3} \, d\theta$

Let,

$5 + \cos 2\theta = p$

$\Rightarrow -2\sin 2\theta = \frac{dp}{d\theta}$

$\therefore \sin 2\theta \, d\theta = \frac{-dp}{2}$

$= \frac{-1}{2} \int \frac{dp}{p^3}$

$= \frac{-1}{2} \frac{p^{-3+1}}{-3+1} + c$

$= \frac{1}{4(5+\cos 2\theta)^2} + c$ Ans:

36. $\int \tan^3 5x \sec^2 5x \, dx$

Let, $\tan 5x = p$

$\Rightarrow \sec^2 5x \, dx = dp$

$= \int p^3 \, dp$

$= \frac{p^4}{4} + c$

$= \frac{1}{4} \tan^4 5x + c$ Ans:

37. $\int \frac{\sec^2 x\, dx}{\sqrt{1-\tan^2 x}}$

Let, ~~$\tan^2 x = p$~~

$\Rightarrow$ ~~sec~~ $\tan x = p$

$\Rightarrow \sec^2 x\, dx = dp$

$= \int \frac{dp}{\sqrt{1-p^2}}$

$= \sin^{-1} p + C$

$= \sin^{-1}(\tan x) + C$ Ans:

38. $\int \frac{\sin\theta\, d\theta}{\cos^2\theta + 1}$

Let, $\cos\theta = p$

$\Rightarrow \sin\theta\, d\theta = -dp$

$= \int \frac{-dp}{p^2+1}$

$= -\tan^{-1} p + C$

$= -\tan^{-1}(\cos\theta) + C$ Ans:

39. $\int \sec^3 2x \tan 2x\, dx$

Let, $\sec 2x = p$

$\Rightarrow 2\sec 2x \tan 2x = \frac{dp}{dx}$

$\Rightarrow \sec 2x \tan 2x\, dx = \frac{dp}{2}$

$= \frac{1}{2}\int p^2 dp$

$= \frac{1}{2}\cdot\frac{1}{3}p^3 + C$

$= \frac{1}{6}\sec^3 2x + C$ Ans:

40. $\int [\sin(\sin\theta)] \cos\theta\, d\theta$

Let, $\sin\theta = p$

$\therefore \cos\theta\, d\theta = dp$

$= \int \sin p\, dp$

$= -\cos p + C$

$= -\cos(\sin\theta) + C$ Ans:

Ex - 6.4

29. $f(x) = 3x+1$; $a=2$, $b=6$.

Sol^n: $n=4$, $\Delta x = \frac{b-a}{n} = \frac{6-2}{4} = 1$.

(a) Left end point:

$$x^*_k = a + (k-1)\Delta x$$
$$= 2 + (k-1)\cdot 1$$
$$= k+1.$$

$$\therefore f(x^*_k) = 3(k+1)+1 = 3k+4.$$

$$A = \sum_{k=1}^{4} (3k+4)\Delta x = 3\sum_{k=1}^{4} k + 4\sum_{k=1}^{4} 1.$$

$$= 3 \times \frac{4(4+1)}{2} + 4\times 4 = 30+16$$

$$= 46.$$

(b) Right end point: $x^*_k = a + k\cdot\Delta x = 2+k$

$$f(x^*_k) = 3(2+k)+1 = 3k+7.$$

$$A = \sum_{k=1}^{4} f(x^*_k)\cdot \Delta x$$

$$= \sum_{k=1}^{4} (3k+7)\cdot 1$$

$$= 3\sum_{k=1}^{4} k + 7\cdot\sum_{k=1}^{4} 1.$$

$$= 3\,\frac{4(4+1)}{2} + 7\times 4$$

$$= 30+28$$

$$= 58.$$

(c) Mid point : $x^*_k = a(k - 1/2)\cdot \Delta x$

$= 2\,(k - 1/2)\cdot 1$

$= 2k - 1.$

$f(x^*_k) = 3(2k-1)+1 = 6k-2.$

$$A = \sum_{k=1}^{4} f(x^*_k)\cdot \Delta x$$

$$= \sum_{k=1}^{4} (6k-2)\cdot 1$$

$$= 6\sum_{k=1}^{4} k - 2\sum_{k=1}^{4} 1.$$

$$= 6\cdot \frac{4(4+1)}{2} - 2\cdot 4.$$

$$= 60 - 8$$

$$= 52.$$

Ans:

30. $f(x) = 1/x$; $a=1$, $b=9$.

Soln : $A = \sum_{k=1}^{4} f(x^*_k)\cdot \Delta x$

$\Delta x = \frac{9-1}{4} = 2$; $f(x^*_k) = \frac{1}{x^*_k}$

(a) Left end point, $x^*_k = a+(k-1)\Delta x$

$= 1+(k-1)\,2$

$= 2k - 1$

$\therefore f(x^*_k) = \frac{1}{2k-1}$

$$A = \sum_{k=1}^{4} f(x^*_k)\,\Delta x$$

$$= \sum_{k=1}^{4} \frac{1}{2k-1} \times 2.$$

$$= \frac{2}{2\times1-1} + \frac{2}{2\times2-1} + \frac{2}{2\times3-1} + \frac{2}{2\times4-1}$$

$$= 2 + 2/3 + 2/5 + 2/7$$

$$= \frac{352}{105}$$

(b) Right end point, $x^*_k = a + k\Delta x = 1+2k$.

$$\therefore f(x^*_k) = \frac{1}{1+2k}.$$

$$A = \sum_{k=1}^{4} f(x^*_k)\,\Delta x.$$

$$= \sum_{k=1}^{4} \frac{1}{1+2k} \times 2$$

$$= \frac{2}{1+2\times1} + \frac{2}{1+2\times2} + \frac{2}{1+2\times3} + \frac{2}{1+2\times4}$$

$$= \frac{2}{3} + \frac{2}{5} + \frac{2}{7} + \frac{2}{9}$$

$$= \frac{1480}{945} = 296/189.$$

(c) Mid point, $x^*_k = a(k-1/2)\cdot\Delta x$.

$$= 1(k-1/2)\cdot 2 = 2k-1$$

$$A = \sum_{k=1}^{4} f(x^*_k)\,\Delta x$$

$$= \sum_{k=1}^{4} \frac{1}{2k-1} \times 2 \quad = \frac{2}{1-1} + \frac{2}{2-1} + \frac{2}{3-1} + \frac{2}{4-1}$$

$$= \frac{15+2}{3} = \frac{17}{3}$$

(c) Right point : $x^*_k = a + k\Delta x = \pi/4\, k$

$$A = \sum_{k=1}^{4} f(x^*_k)\cdot \Delta x$$

$$= \sum_{k=1}^{4} \cos(\pi/4\, k)\cdot \pi/4$$

$$= \cos\pi/4 \cdot \pi/4 + \cos(\pi/4\cdot 2)\,\pi/4 + \cos(\pi/4\cdot 3)\,\pi/4 + \cos(\pi/4\cdot 4)\,\pi/4.$$

$$= \frac{1}{\sqrt{2}}\cdot \pi/4 + 0 - \frac{1}{\sqrt{2}}\cdot \pi/4 - 1\cdot \pi/4.$$

$$= -\pi/4.$$

Ans:

32. $f(x) = 2x - x^2$; $a = -1$, $b = 3$.

Soln: $\Delta x = \dfrac{3+1}{4} = 1$

(a) Left point, $x^*_k = a + (k-1)\Delta x$

$$= -1 + (k-1)\cdot 1$$

$$= -1 + k - 1$$

$$= k - 2.$$

$$\therefore \sum_{k=1}^{4} f(x^*_k)\cdot\Delta x = \sum_{k=1}^{4}\{2(k-2) - (k-2)^2\}\cdot 1$$

$$= \sum_{k=1}^{4}(2k-4) - \sum_{k=1}^{4}(k^2 - 4k + 4).$$

$$= \{(2\times 1 - 4) + (2\times 2 - 4) + (2\times 3 - 4) + (2\times 4 - 4)\} - \{(1 - 4 + 4) + (2^2 - 4\times 2 + 4) + (3^2 - 4\times 3 + 4) + (4^2 - 4\times 4 + 4)\}$$

$$= (-2 + 0 + 2 + 4) - (1 - 8 + 1 + 4)$$

$$= 4 + 2 = 6$$

(b) Right end point, $x^*_k = a+k\Delta x$

$= 1+k\times 1 = 1+k$

$f(x^*_k) = 2(1+k) - (1+k)^2$

$= 2+k-1-2k-k^2$

$= 1-k-k^2.$

$\therefore \sum_{k=1}^{4} f(x^*_k)\Delta x = \sum_{k=1}^{4} (1-k-k^2).1.$

$= (1-1-1^2)+(1-2-2^2)+(1-3-3^2)+(1-4-4^2)$

$= -1-5-11-19$

$= -36$

(c) Mid end point, $x^*_k = a(k-1/2)\Delta x$

$= 1(k-1/2)\cdot 1.$

$= k-1/2.$

$f(x^*_k) = 2(k-1/2)-(k-1/2)^2$

$= 2k-1-k^2+k-1/4$

$= 3k-k^2-5/4$

$\therefore \sum_{k=1}^{4} f(x^*_k)\Delta x = \sum_{k=1}^{4} (3k-k^2-5/4)\cdot 1.$

$= (3-1^2-5/4)+(3\times 2-2^2-5/4)+(3\times 3-3^2-5/4)+(3\times 4-4^2-5/4)$

$= \frac{3}{4}+\frac{3}{4}-\frac{5}{4}-\frac{21}{4}$

$= -20/4 = -5$

Ans:

38. $y = 5 - x$; $a = 0$, $b = 5$.

Sol^n: $\Delta x = \frac{5-0}{n} = 5/n$.

Right point: $x^*_k = a + k\Delta x$

$= 0 + k \cdot 5/n$

$= 5k/n$.

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x^*_k)\Delta x$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} (5 - 5k/n)5/n.$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \left(5 \cdot \frac{5}{n} - \frac{5k}{n} \cdot \frac{5}{n}\right)$$

$$= \lim_{n\to\infty} \left\{\frac{25}{n} \sum_{k=1}^{n} 1 - \frac{25}{n^2} \sum_{k=1}^{n} k\right\}$$

$$= \lim_{n\to\infty} \left\{\frac{25}{n} \times n - \frac{25}{2} \cdot \frac{n+1}{n}\right\}$$

$$= \lim_{n\to\infty} \left\{25 - 25/2\,(1 + 1/n)\right\}$$

$$= 25 - 25/2$$

$$= 25/2$$

Ans:

39. $f(x) = 9 - x^2$, $a = 0$, $b = 3$.

Sol^n: $\Delta x = \frac{3-0}{n} = 3/n$.

Right point : $x^*_k = a + k \cdot \Delta x$

$$= 0 + k \cdot 3/n = 3k/n.$$

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x^*_k)\,\Delta x$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \left\{ 9 - (3k/n)^2 \right\} 3/n.$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{27}{n} - \frac{27k^2}{n^3} \right)$$

$$= \lim_{n\to\infty} \left\{ \frac{27}{n} \sum_{k=1}^{n} 1 - \frac{27}{n^3} \sum_{k=1}^{n} k^2 \right\}$$

$$= \lim_{n\to\infty} \left\{ \frac{27}{n} \cdot n - \frac{27}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} \right\}$$

$$= \lim_{n\to\infty} \left\{ 27 - \frac{27}{6} \cdot \frac{2n^2 + n + 2n + 1}{n^2} \right\}$$

$$= \lim_{n\to\infty} \left\{ 27 - \frac{27}{6} \cdot \left(2 + \frac{3}{n} + \frac{1}{n^2} \right) \right\}.$$

$$= 27 - 9$$

$$= 18$$

Ans:

Ex. 6·5

$$= -\int_0^1 f(x)\,dx + \int_0^5 f(x)\,dx$$

$$= 2 + 1$$

$$= 3 \text{ (Ans)}$$

21.(a) $\int_0^1 \left(x + 2\sqrt{1-x^2}\right)dx$

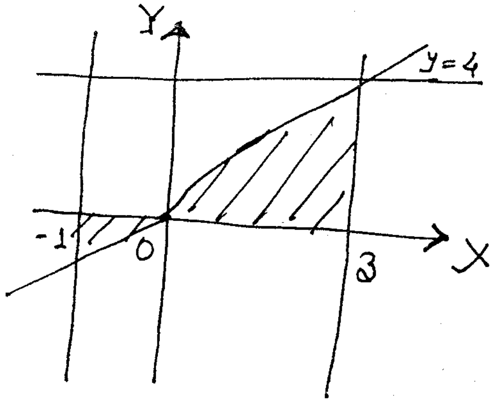
$$= \int_0^1 x\,dx + 2\int_0^1 \sqrt{1-x^2}\,dx$$

$$= \left[\frac{x^2}{2}\right]_0^1 + 2 \times \frac{1}{4}\pi(1)^2$$

$$= \frac{1}{2} + \frac{1}{2}\pi$$

$$= \frac{1+\pi}{2} \text{ (Ans)}$$

(b) $\int_{-1}^{3} (4-5x)\,dx$



$= 4\int_{-1}^{3} dx - 5\int_{-1}^{3} x\,dx$

$= 4[x]_{-1}^{3} - 5\left[\frac{x^2}{2}\right]_{-1}^{3}$

$= 4(3+1) - 5\left(\frac{9}{2} - \frac{1}{2}\right)$

$= 16 - 5 \times 4$

$= -4$ (Ans)

25. $\int_{0}^{10} \sqrt{10x - x^2}\,dx$

$= \int_{0}^{10} \sqrt{(5)^2 - (x-5)^2}\,dx$

$= \frac{1}{2}\pi(5)^2$

$= \frac{25}{2}\pi$ (Ans)

15. $\int_{-\pi/2}^{\pi/2} \sin\theta \, d\theta$

$= -[\cos\theta]_{-\pi/2}^{\pi/2}$

$= -[\cos \pi/2 - \cos(-\pi/2)]$

$= 0$ (Ans)

16. $\int_{0}^{\pi/4} \sec^2\theta \, d\theta$

$= [\tan\theta]_{0}^{\pi/4}$

$= [\tan \pi/4 - \tan 0]$

$= 1$ (Ans)

17. $\int_{-\pi/4}^{\pi/4} \cos x \, dx$

$= [\sin x]_{-\pi/4}^{\pi/4}$

$= \sin \pi/4 - \sin(-\pi/4)$

$= \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}}$

$= \dfrac{2}{\sqrt{2}} = \sqrt{2}$ (Ans)

18. $\int_0^1 (x - \sec x \tan x)\, dx$

$= \int_0^1 x\, dx - \int_0^1 \sec x \tan x\, dx$

$= \left[\dfrac{x^2}{2}\right]_0^1 - \left[\sec x\right]_0^1$

$= \left[\dfrac{1}{2} - \dfrac{0}{2}\right] - \left[\dfrac{1}{\cos 1} - \dfrac{1}{\cos 0}\right]$

$= \dfrac{1}{2} - [1 - 1] = \dfrac{1}{2}$ (Ans)

19. $\int_{\ln 2}^{3} 5e^x\, dx$

$= 5\int_{\ln 2}^{3} e^x\, dx$

$= 5\left[e^x\right]_{\ln 2}^{3}$

$= 5\left[e^3 - e^{\ln 2}\right]$

$= 5e^3 - 10$ (Ans)

~~20~~. $\int_{1/\sqrt{2}}^{1} \frac{1}{2x}\,dx$

$= \frac{1}{2}\left[\ln[x]\right]_{1/\sqrt{2}}^{1}$

$= \frac{1}{2}\left[\ln 1 - \ln 1/\sqrt{2}\right]$

$= \frac{1}{2}\ln\frac{1}{1/\sqrt{2}} \quad = \frac{1}{2}\ln\sqrt{2}$

21. $\int_{0}^{1/\sqrt{2}} \frac{dx}{\sqrt{1-x^2}}$

$= \left[\sin^{-1}x\right]_{0}^{1/\sqrt{2}}$

$= \left[\sin^{-1} 1/\sqrt{2} - \sin^{-1} 0\right]$

$= \pi/4$ (Ans)

22. $\int_{-1}^{1} \frac{dx}{1+x^2}$

$= \left[\tan^{-1}x\right]_{-1}^{1}$

$= \tan^{-1}1 - \tan^{-1}(-1)$

$= \frac{\pi}{4} + \frac{\pi}{4}$

$= \frac{2\pi}{4} = \pi/2$ (Ans)

$= \sin \pi/4 - \sin(-\pi/4)$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$= \frac{2}{\sqrt{2}} = \sqrt{2}$ (Ans)

18. $\int_0^1 (x - \sec x \tan x)\, dx$

$= \int_0^1 x\, dx - \int_0^1 \sec x \tan x\, dx$

$= \left[\frac{x^2}{2}\right]_0^1 - \left[\sec x\right]_0^1$

$= \left[\frac{1}{2} - \frac{0}{2}\right] - \left[\frac{1}{\cos 1} - \frac{1}{\cos 0}\right]$

$= \frac{1}{2} - [1 - 1] = \frac{1}{2}$ (Ans)

19. $\int_{\ln 2}^{3} 5e^x\, dx$

$= 5\int_{\ln 2}^{3} e^x\, dx$

$= 5\left[e^x\right]_{\ln 2}^{3}$

$= 5\left[e^3 - e^{\ln 2}\right]$

$= 5e^3 - 10$ (Ans)

$\cancel{23}$. $\int_{\sqrt{2}}^{2} \frac{dx}{x\sqrt{x^2-1}}$

Let,

$x = \sec\theta$

$dx = \sec\theta \tan\theta \, d\theta$

$\int \frac{dx}{x\sqrt{x^2-1}} = \int \frac{\cancel{\sec\theta}\tan\theta \, d\theta}{\cancel{\sec\theta}\sqrt{\sec^2\theta - 1}}$

$= \int \frac{\cancel{\tan\theta}\, d\theta}{\cancel{\tan\theta}}$

$= \int d\theta = \theta$

$\sec\theta = x \therefore \cos\theta = 1/x$

$\Rightarrow \theta = \cos^{-1} 1/x$

$= \left[\cos^{-1} 1/x\right]_{\sqrt{2}}^{2}$

$= \left[\cos^{-1} 1/2 - \cos^{-1} 1/\sqrt{2}\right]$

$= \cos^{-1}\cos\pi/3 - \cos^{-1}\cos\pi/4$

$= \pi/3 - \pi/4 = \frac{4\pi - 3\pi}{12} = \frac{\pi}{12}$ (Ans)

$\cancel{24}$. $\int_{-\sqrt{2}}^{-2/\sqrt{3}} \frac{dx}{x\sqrt{x^2-1}}$.

$= \left[\cos^{-1} 1/x\right]_{-\sqrt{2}}^{-2/\sqrt{3}}$

$= \left[\cos^{-1}\left(\frac{1}{-\sqrt{2}}\right) - \cos^{-1}\left(1/{-2/\sqrt{3}}\right)\right]$

$= \left[\cos^{-1}\cos\pi/4 - \cos^{-1}\cos\pi/6\right]$

$= \pi/4 - \pi/6$

$= \frac{3\pi - 2\pi}{12} = \pi/12$ (Ans)

25. $\int_1^4 \left(\frac{3}{\sqrt{t}} - 5\sqrt{t} - t^{-3/2}\right)dt$

$= \int_1^4 3t^{-1/2}dt - 5\int_1^4 t^{1/2}dt - \int_1^4 t^{-3/2}dt$

$= 3\times 2\,\left[t^{1/2}\right]_1^4 - 5\times\frac{2}{3}\,\left[t^{3/2}\right]_1^4 + 2\left[t^{-1/2}\right]_1^4$

$= 6\,\left[\sqrt{4} - \sqrt{1}\right] - \frac{10}{3}\left[2^{2\times 3/2} - 1^{3/2}\right] + 2\left[t^{-1/2}\right]_1^4$

$= 6 - \frac{10}{3}\cdot\times 7 + 2\left[4^{-1/2} - 1^{-1/2}\right]$

$= 6 - \frac{70}{3} + 2\cancel{t^{-1/2}}\left(2^{2\times -1/2} - 1^{-1/2}\right)$

$= \frac{-52}{3} + \cancel{2t^{-1/2}}\ 2\times\frac{1}{\sqrt{2}} - 2$

$= -\frac{52}{3} + \sqrt{2} - 2$